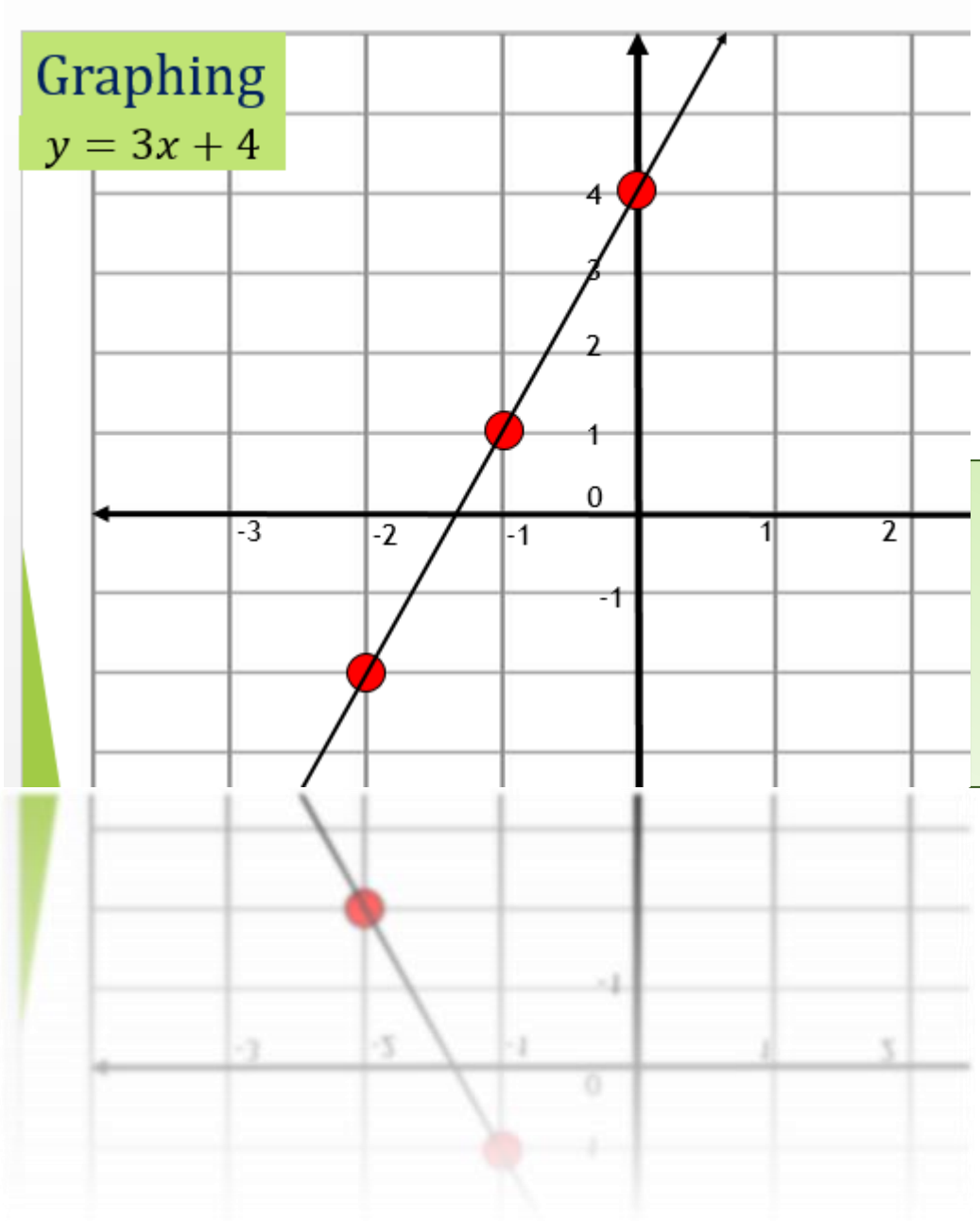


Equations of Lines

In the Coordinate Plane

Sections 3-7 and 3-8

We are focusing on two types of equations: point-slope and slope-intercept

Point-slope form:

- $y - y_1 = m(x - x_1)$
- Best used with slope and a point
- Can also be used with two points
- Slope can be found from two points

Slope-intercept form:

- $y = mx + b$
- Best for graphing lines
- Can be used to plot y-intercept
- Can be derived from a graph

Examples

- Given a slope of 3 and point (2,-1) create an equation of a line using the point-slope form
  - Step 1: write template:

    $y - y_1 = m(x - x_1)$
  - Step 2: substitute given information:

    $y - (-1) = 3(x - 2)$
  - Step 3: combine double negatives

    $y + 1 = 3(x - 2)$
- Given a slope of 4 and a y-intercept of -2 create an equation of a line using the slope-intercept form
  - Step 1: write the template:

    $y = mx + b$
  - Step 2: substitute the given information:

    $y = 4x - 2$

Examples (continued)

- Given a slope of -6 and point (7,10) create an equation of a line using the point-slope form
  - Step 1: write template:

    $y - y_1 = m(x - x_1)$
  - Step 2: substitute given information:

    $y - 10 = -6(x - 7)$
  - Step 3: combine double negatives

    None!
- Given a slope of 7/3 and a y-intercept of 5 create an equation of a line using the slope-intercept form
  - Step 1: write the template:

    $y = mx + b$
  - Step 2: substitute the given information:

    $y = (7/3)x + 5$

Try it!

- Given a slope of 4 and point (-2,3) create an equation of a line using the point-slope form
  - Step 1: what is the format?
  - Step 2: what do we substitute?
  - Step 3: are we done yet?
- Given a slope of -5 and y-intercept 7 create an equation of a line using the slope intercept form
  - Step 1: what is the format?
  - Step 2: what do we substitute?
  - Step 3: are we done?

Converting from point-slope to slope-intercept

It's all about simplifying!

- To convert just simplify the point-slope format
  - $y - 5 = 2(x + 4)$
  - $y - 5 = 2x + 8$ (distribute the slope value)
  - $y = 2x + 13$ (add five to both sides)

  - $y + 2 = -4(x - 7)$
  - $y + 2 = -4x + 28$ (distribute the slope)
  - $y = -4x + 26$ (subtract two)

  - $y + 6 = (1/2)(x + 7)$
  - $y + 6 = (1/2)x + 7/2$ (distribute the slope)
  - $y = (1/2)x + 7/2 - 12/2$ (subtract 6)
  - $y = (1/2)x - 5/2$

Computing Slope

It's just the change in y over the change in x

- If we do not know the slope we can either derive it from the graph (by counting squares) or, if we know two points on the line, computing it using the point-slope format
- The usual format is

$$y - y_1 = m(x - x_1)$$

But we can also write it as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- Try it with (2,-1) and (4, 3)

$$m = \frac{3 - (-1)}{4 - 2} = \frac{4}{2} = 2$$

Computing Slope

Using it to create the equation of a line

- Once we have found the slope we just take one (either one!) of the given points and substitute it into the point-slope format along with the slope that we just found
- The format is

$$y - y_1 = m(x - x_1)$$

We found the slope was 2 for points (2, -1) and (4,3). Now just pick one of the points and substitute it into the equation along with the slope:

$$y - 3 = 2(x - 4)$$

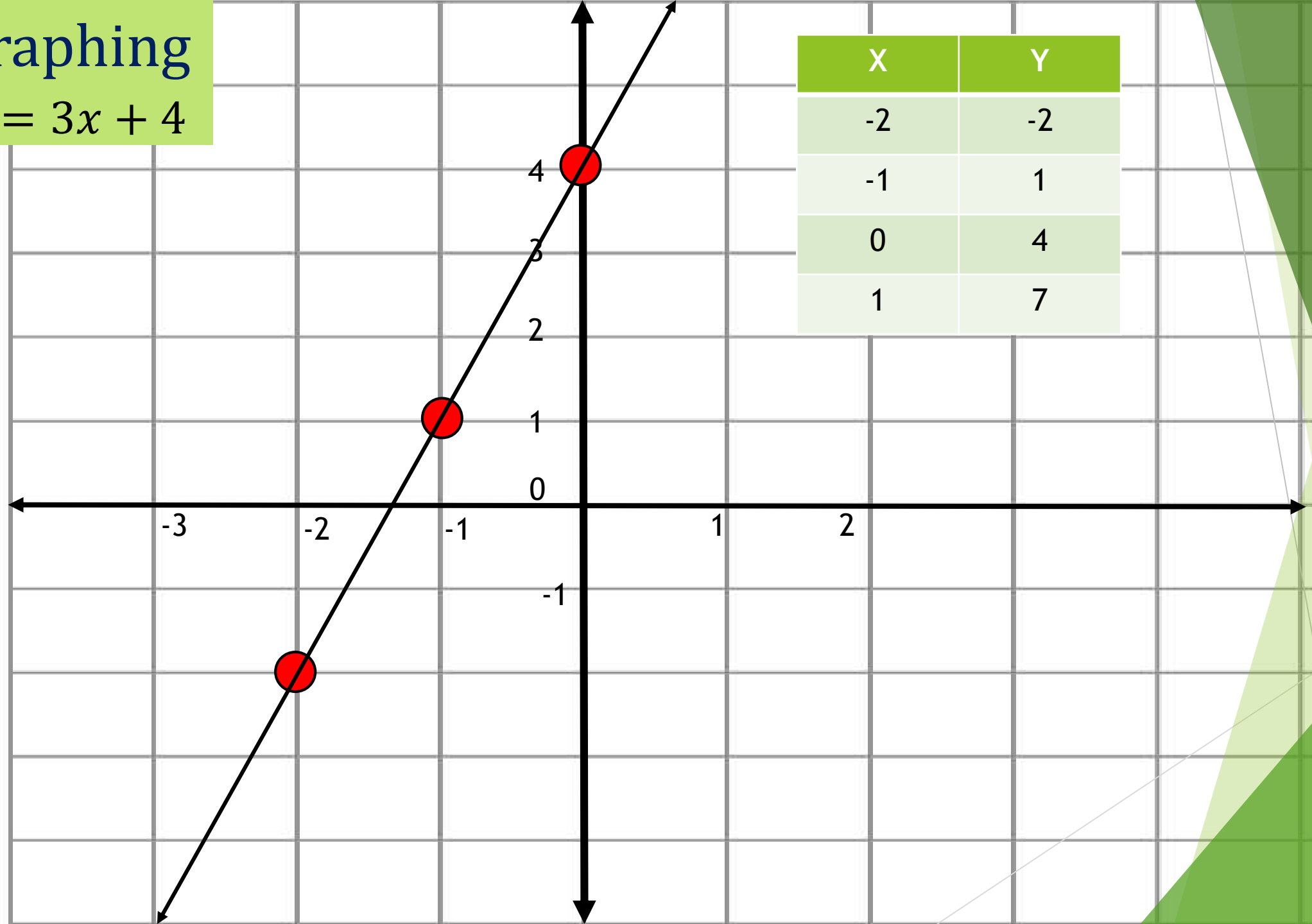
And we're done!

- Question: does it matter which point we use? Answer: no!

Using (4,3)	Using (2,-1)
$y - 3 = 2(x - 4)$	$y - (-1) = 2(x - 3)$
$y - 3 = 2x - 8$	$y + 1 = 2x - 6$
$y = 2x - 5$	$y = 2x - 5$

Graphing

$y = 3x + 4$

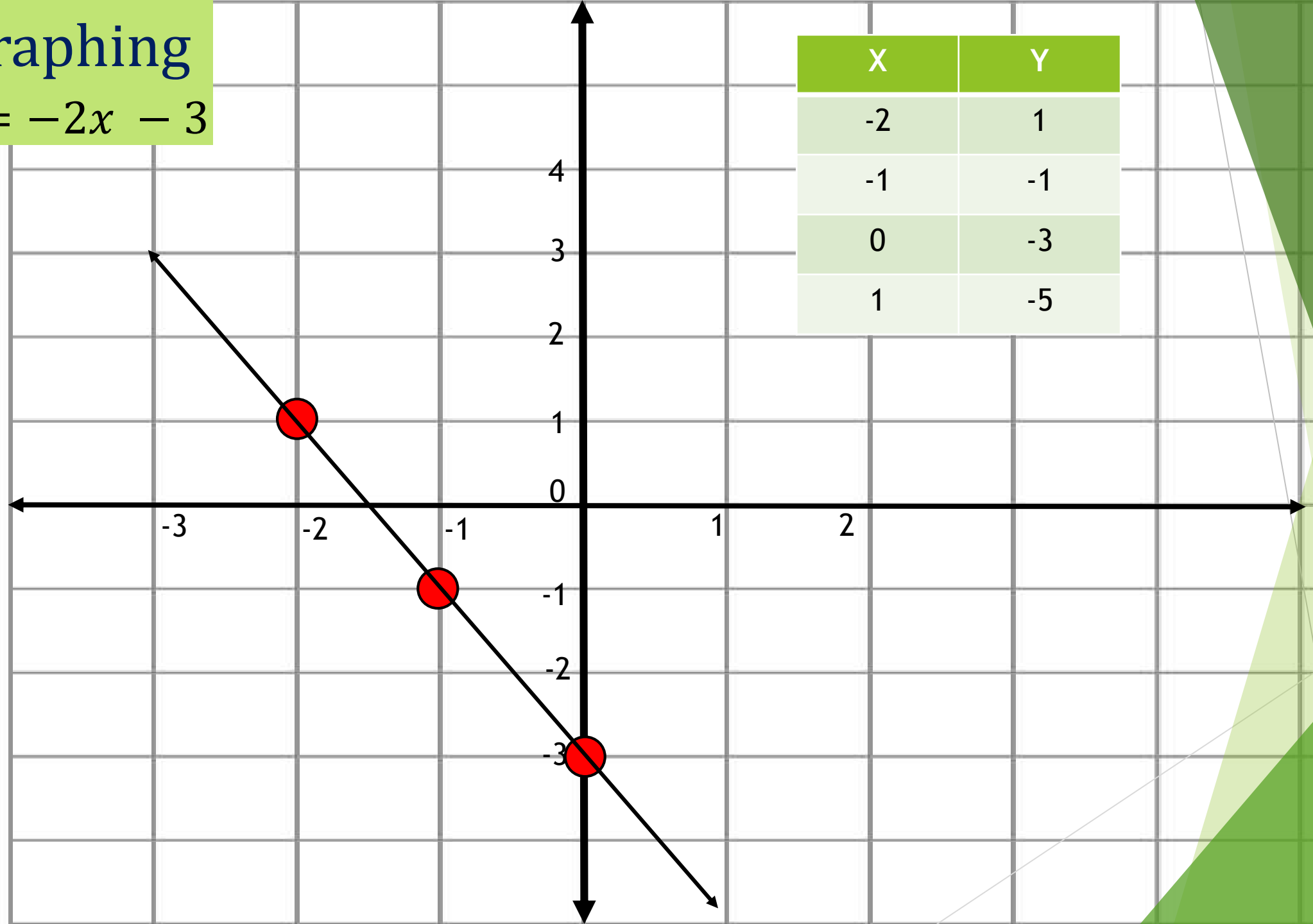


X	Y
-2	-2
-1	1
0	4
1	7

Graphing

$y = -2x - 3$

X	Y
-2	1
-1	-1
0	-3
1	-5



Slopes of parallel and perpendicular lines

One is the same, one is the opposite inverse!

- Since parallel lines have the same slope, the slope of one line, m_1, has to be equal to the slope of a second line, m_2, for the lines to be parallel:

$$\mathbf{m_1 = m_2}$$

- Perpendicular lines will have slopes that are *opposite inverses* of each other. Opposite means the sign is the opposite of the original and inverse means the slope, expressed as a fraction, is "flipped".

$$\mathbf{m_2 = -\frac{1}{m_1}}$$

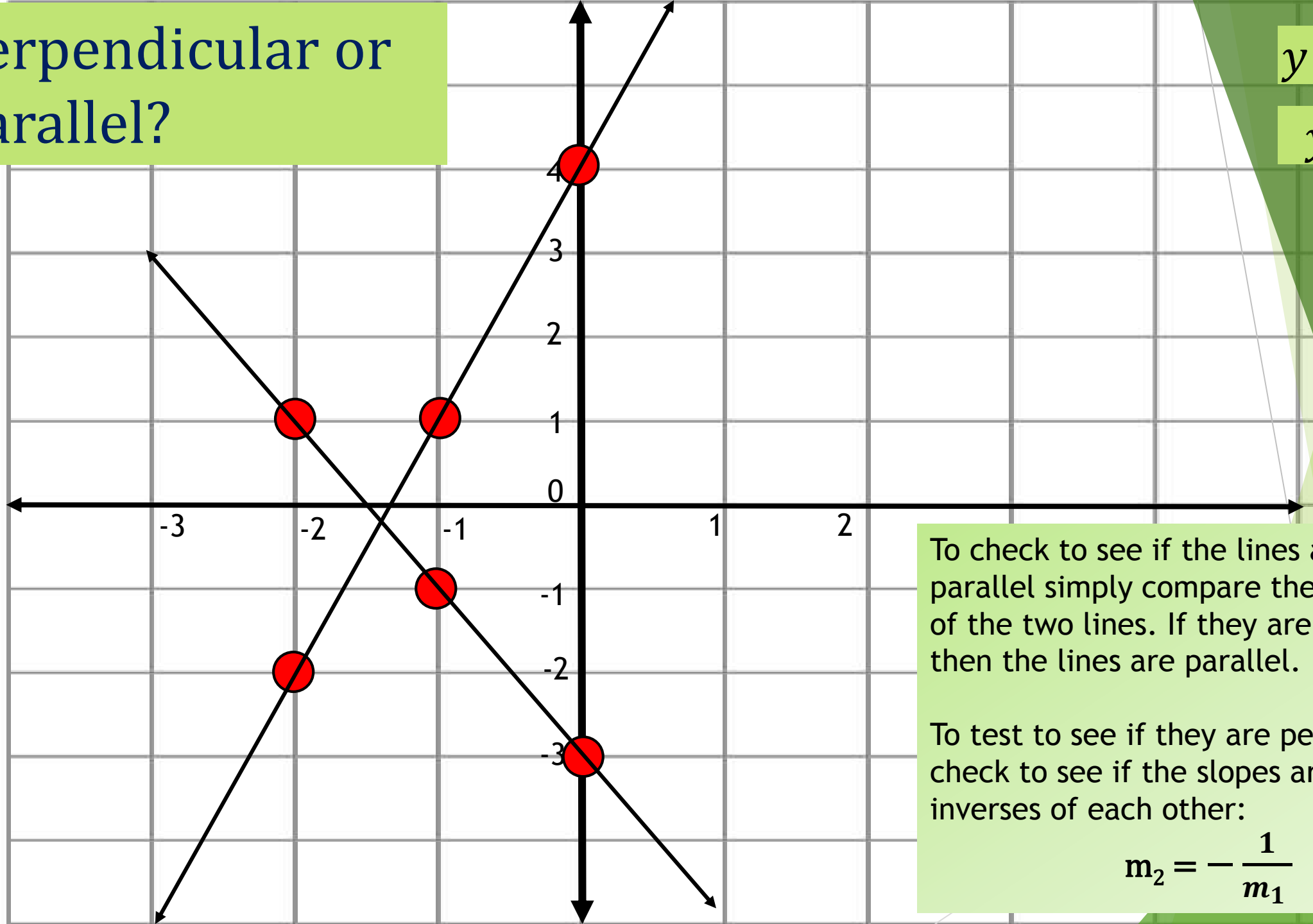
If $m_1 = 4$ then m_2 is:

$$\mathbf{-\frac{1}{m_1} = -\frac{1}{4}}$$

Perpendicular or Parallel?

$y = -2x - 3$

$y = 3x + 4$

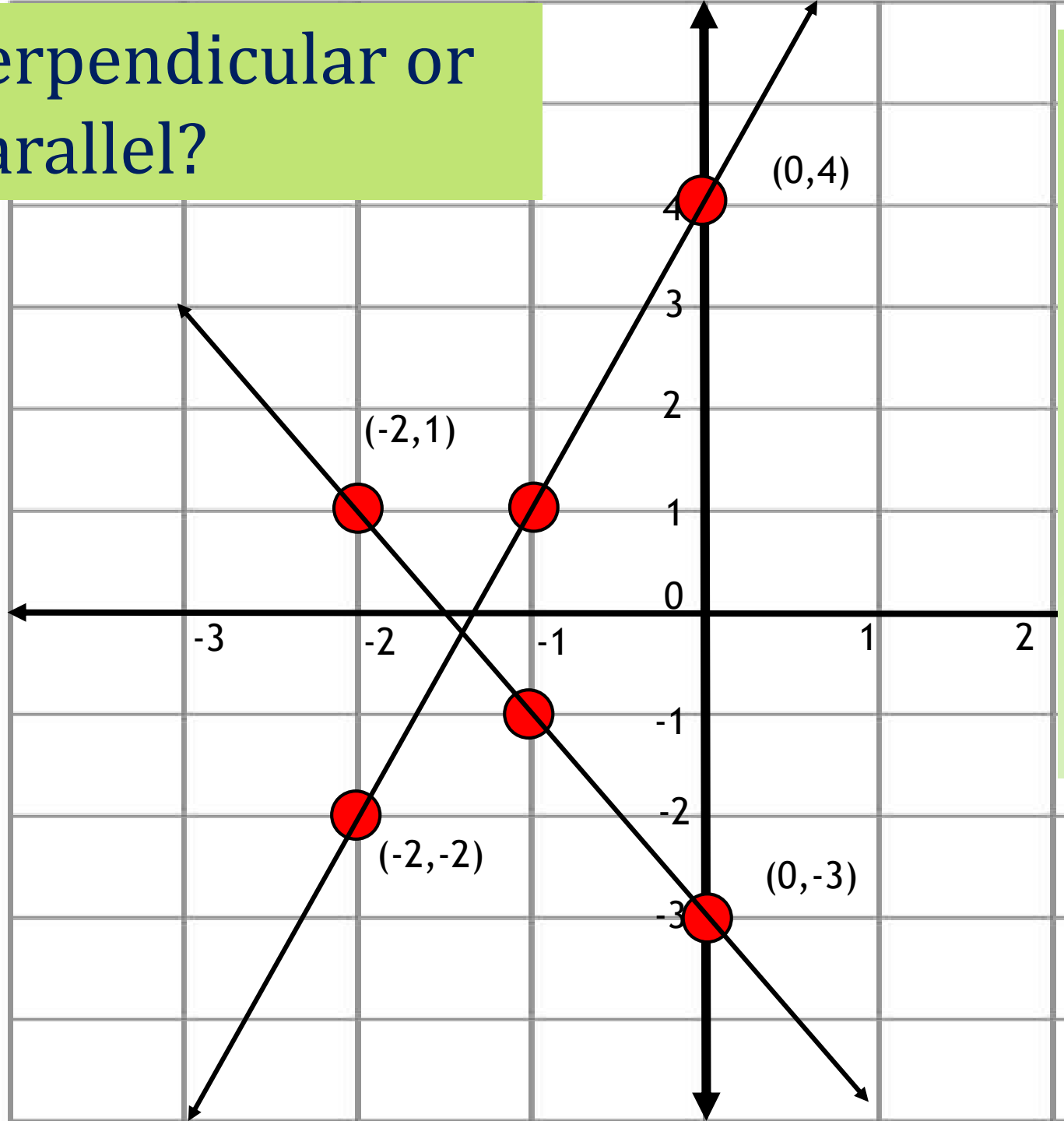


To check to see if the lines are parallel simply compare the slopes of the two lines. If they are the same then the lines are parallel.

To test to see if they are perpendicular check to see if the slopes are opposite inverses of each other:

$$\mathbf{m_2 = -\frac{1}{m_1}}$$

Perpendicular or Parallel?



If you have points instead of the equations of a line use the point-slope formula to compute the slope(s).

$$\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{-3 - 1}{0 - (-2)} = \frac{-4}{2} = -2$$

$$m_2 = \frac{4 - (-2)}{0 - (-2)} = \frac{6}{2} = 3$$

Since -2 is not equal to $-\frac{1}{3}$ the lines are not perpendicular